

Topics : Surface Tension, Elasticity & Viscosity, Geometrical Optics, Circular Motion, Rigid Body Dynamics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.5

(3 marks, 3 min.)

M.M., Min.

[15, 15]

Subjective Questions ('-1' negative marking) Q.6

(4 marks, 5 min.)

[4, 5]

Comprehension ('-1' negative marking) Q.7 to Q.9

(3 marks, 3 min.)

[9, 9]

1. A capillary tube with inner cross-section in the form of a square of side a is dipped vertically in a liquid of density ρ and surface tension σ which wet the surface of capillary tube with angle of contact θ . The approximate height to which liquid will be raised in the tube is : (Neglect the effect of surface tension at the corners capillary tube)

(A) $\frac{2\sigma\cos\theta}{\rho g}$

(B) $\frac{4\sigma\cos\theta}{\rho g}$

(C) $\frac{8\sigma\cos\theta}{\rho g}$

(D) None of these

2. Four uniform wires of the same material are stretched by the same force. The dimensions of wire are as given below. The one which has the minimum elongation has :

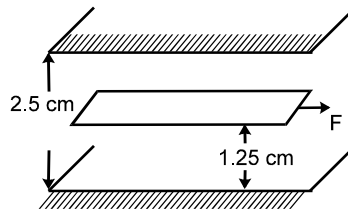
(A) radius 3mm, length 3m

(B) radius 0.5 mm, length 0.5 m

(C) radius 2mm, length 2m

(D) radius 3mm, length 2m

3. A space 2.5 cm wide between two large plane surfaces is filled with oil. Force required to drag a very thin plate of area 0.5 m^2 just midway the surfaces at a speed of 0.5 m/sec is 1N . The coefficient of viscosity in kg-s/m^2 is :



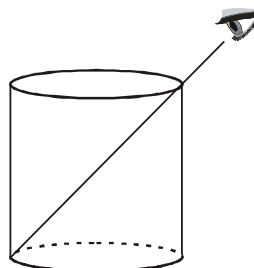
(A) 5×10^{-2}

(B) 2.5×10^{-2}

(C) 1×10^{-2}

(D) 7.5×10^{-2}

4. A glass beaker has diameter 4cm wide at the bottom. An observer observes the edge of bottom when beaker is empty as shown in figure. When the beaker is completely filled with liquid of refractive index $n = \sqrt{5/2}$, he can just see the centre of bottom, then the height of glass beaker is :



(A) 4 cm

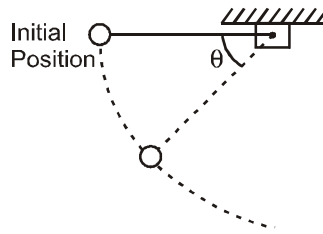
(B) $\sqrt{5/2}$ cm

(C) 16 cm

(D) None of these



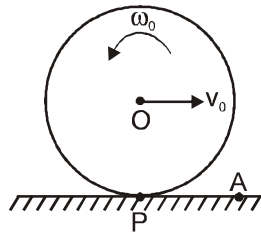
5. The given figure shows a small mass connected to a string, which is attached to a vertical post. If the mass is released from rest when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is



- (A) $2g \sin\theta$ (B) $2g \cos\theta$ (C) $g\sqrt{3\cos^2\theta + 1}$ (D) $g\sqrt{3\sin^2\theta + 1}$
6. A 40 cm long wire having a mass 3.2 gm and area of cross section 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode, it vibrates with a frequency $1000/64 \text{ Hz}$. Find the young's modulus of the wire in the form $X \times 10^8 \text{ N/m}^2$ and fill value of X.

COMPREHENSION

A uniform wheel is released on a rough horizontal floor after imparting it an initial horizontal velocity v_0 and angular velocity ω_0 as shown in the figure below. Point O is the centre of mass of the wheel and point P is its instantaneous point of contact with the ground. The radius of wheel is r and its radius of gyration about O is k . Coefficient of friction between wheel and ground is μ . A is a fixed point on the ground.



7. Which of the following is conserved ?
 (A) linear momentum of wheel
 (B) Angular momentum of wheel about O
 (C) Angular momentum of wheel about A
 (D) none of these
8. If the wheel comes to permanent rest after sometime, then :
 (A) $v_0 = \omega_0 r$ (B) $v_0 = \frac{\omega_0 k^2}{r}$ (C) $v_0 = \frac{\omega_0 r^2}{R}$ (D) $V_0 = \omega_0 \left(r + \frac{k^2}{r} \right)$
9. In above question, distance travelled by centre of mass of the wheel before it stops is -
 (A) $\frac{v_0^2}{2\mu g} \left(1 + \frac{r^2}{k^2} \right)$ (B) $\frac{v_0^2}{2\mu g}$ (C) $\frac{v_0^2}{2\mu g} \left(1 + \frac{k^2}{r^2} \right)$ (D) None of these



Answers Key

1. (B) 2. (D) 3. (B) 4. (A)
 5. (D) 6. $1 \times 10^9 \text{ N/m}^2$ 7. (C)
 8. (B) 9. (B)

Hints & Solutions

1. Upward force by capillary tube on top surface of liquid is

$$f_{\text{up}} = 4\sigma a \cos \theta$$

If liquid is raised to a height h then we use

$$4\sigma a \cos \theta = ha^2 \rho g$$

$$\text{or } h = \frac{4\sigma \cos \theta}{a\rho g} \quad \text{Ans.}$$

$$2. \Delta l = \frac{F\ell}{\pi r^2 y} \Rightarrow \Delta l \propto \frac{\ell}{r^2}$$

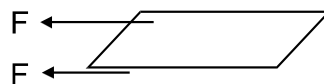
Only option 'radius 3mm, length 2m' is satisfying the above relation.

$$3. \text{Velocity gradient} = \frac{0.5}{\frac{2.5}{2} \times 10^{-2}}$$

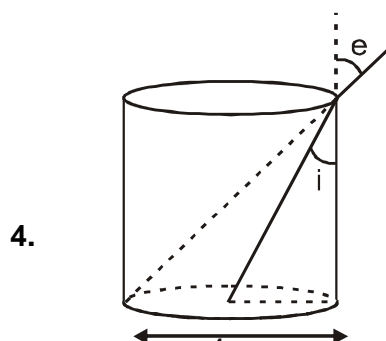
as force on the plate due to viscosity is from upper as well as lower portion of the oil, equal from each part,

$$\text{Then, } F = 2\eta A \frac{dv}{dz}$$

$$= 2 \times \eta \times (0.5) \frac{0.5}{1.25 \times 10^{-2}}$$

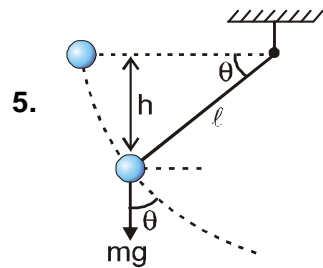


$$\Rightarrow \eta = 2.5 \times 10^{-2} \text{ kg - sec/m}^2$$



$$\sqrt{\frac{5}{2}} \sin i = 1 \sin e \Rightarrow \sqrt{\frac{5}{2}} \frac{2 \text{ cm}}{\sqrt{4 \text{ cm}^2 + h^2}}$$

$$= 1 \times \frac{4 \text{ cm}}{\sqrt{16 \text{ cm}^2 + h^2}} \Rightarrow h = 4 \text{ cm}$$



From conservation of energy

$$mgh = \frac{1}{2} mv^2$$

$$\Rightarrow mg\ell \sin\theta = \frac{1}{2} mv^2$$

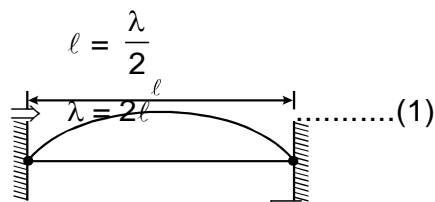
$$\Rightarrow 2g \sin\theta = \frac{v^2}{\ell} = a_c$$

$$g \cos\theta = a_t$$

$$\text{Total acceleration } a = \sqrt{a_c^2 + a_t^2}$$

$$= g\sqrt{\cos^2\theta + (2\sin\theta)^2} = g\sqrt{3\sin^2\theta + 1}$$

6. $\mu = \frac{3.2 \text{ gm}}{40 \text{ cm}} = \frac{3.2 \times 10^{-3}}{40 \times 10^{-2}} = \frac{3.2}{40} = \frac{32}{4000} \text{ kg/m}$



$$f = \frac{v}{\lambda} = \frac{1}{2l} \sqrt{\frac{T}{\mu}}$$

$$\Rightarrow \frac{1000}{64} = \frac{1}{2 \times 40 \times 10^{-2}} \sqrt{\frac{T}{32/4000}}$$

$$\Rightarrow \left[\frac{1000}{64} \times 2 \times 40 \times 10^{-2} \right]^2 \frac{32}{4000} = T$$

$$\frac{1000}{64} \times \frac{32}{4000} = T$$

$$\Rightarrow T = \frac{10}{8} \text{ N}$$

$$\text{अब } y = \frac{\frac{10/8}{10^{-6}}}{\frac{.05 \times 10^{-2}}{40 \times 10^{-2}}} = \frac{10^7}{8} \frac{40}{(.05)}$$

$$= 10^9 \text{ N/m}^2. \quad [\text{Ans. } 1 \times 10^9 \text{ N/m}^2]$$

7. Torque of friction about A is zero.

8. Angular momentum conservation about point A.

$$L_{\text{in}} = mv_0 r - mk^2 \omega_0$$

$$L_{\text{fin}} = 0$$

$$L_{\text{fin}} = L_{\text{in}}$$

$$\Rightarrow v_0 = \omega_0 k^2 / r.$$

9. $a_{\text{cm}} = -\mu g$

$$0^2 = v_0^2 - 2\mu g s \quad \Rightarrow S = \frac{v_0^2}{2\mu g}$$

